The First African Conference on High Energy Physics
Rabat-Salé-Kénitra Regional University Consortium, Morocco, October 23-27, 2023

Neutrino oscillations in external environment

N.Dolganov[1], **A.Popov**[1], **V.Shakhov**[1], **K.Stankevich**[1], **A.Studenikin**[1,2]
1. Faculty of Physics, Lomonosov Moscow State University, Moscow 119991, Russia
2. National Centre for Physics and Mathematics, 607328 Satis, Nizhny Novgorod District, Russia
dolganov.nm18@physics.msu.ru; ar.popov@physics.msu.ru; schakhov.vv15@physics.msu.ru; kostan.10@mail.ru; studenik@srd.sinp.msu.ru

Neutrino effective Hamiltonian

We describe neutrino flavour, spin and spin-flavour oscillations engendered by neutrino interactions with an external electric current due to neutrino charge radii and anapole moments. We consider two flavour neutrinos with two possible helicities $\nu_f = (\nu_e^-, \nu_\alpha^-, \nu_e^+, \nu_\alpha^+)$, $\alpha = \mu, \tau$ and perform calculations that are analogous to those in [1, 2, 3]. In the mass basis the neutrino effective potential describing electromagnetic interactions of the neutrino field ν with the external electric current is given by

$$H_J^{(m)fi} = \lim_{q\to 0} \frac{1}{T} \frac{\langle \nu_f(p_f, h_f) | \int d^4x \mathcal{H}_J |\nu_i(p_i, h_i)\rangle}{\langle \nu(p,h)|\nu(p,h)\rangle}, \quad (1)$$

where $q = p_i - p_f$, p_i and p_f are the initial and final neutrino momenta, T is the normalization time. The matrix element

$$\langle \nu_f(p_f, h_f) | \mathcal{H}_J |\nu_i(p_i, h_i)\rangle = \overline{u}_f(p_f, h_f)\Lambda_\mu^{fi}(q)\frac{1}{q^2}u_i(p_i, h_i)J^\mu_{EM}e^{-iqx}. \quad (2)$$

is dertermined by the electric current of the external charged fermions f (the protons or electrons), $J^\mu_{EM} = e(n_f, n_f\mathbf{v}_f)$. Note that Λ_μ^{fi} in (2) contains the corresponding terms of the neutrino electromagnetic vertex (for its decomposition see [4]). We are interested only in the electric charge and anapole form factors

$$\Lambda_\mu^{fi}(q) = (\gamma_\mu - q_\mu\gamma_\nu q^\nu/q^2)\left[f_Q^{fi}(q^2) + f_A^{fi}(q^2)q^2\gamma_5\right], \quad (3)$$

where $f_Q^{fi}(q^2)$ and $f_A^{fi}(q^2)$ are charge and anapole form factors in the mass basis. The neutrino charge radius is determined by the second term in the expansion of the neutrino charge form factor $f_Q(q^2) = f_Q(0) + q^2 \left.\frac{df_Q(q^2)}{dq^2}\right|_{q^2=0} + \ldots$ and the charge radius is given by $\langle r^2\rangle = 6 \left.\frac{df_Q(q^2)}{dq^2}\right|_{q^2=0}$.
We consider the case of zero neutrino millicharge $f_Q(0) = 0$. Therefore, the electromagnetic vertex accounted for the charge radius r and the anapole moment $a = f_A(0)$ reads

$$\Lambda_\mu^{fi}(q) = (q^2\gamma_\mu - q_\mu\gamma_\nu q^\nu)\left[\frac{\langle r^2\rangle^{fi}}{6} + a^{fi}\gamma_5\right]. \quad (4)$$

This vertex gives the following effective interaction Hamiltonian

$$H_J^{(m)fi} = \frac{1}{\sqrt{E_f E_i}}\overline{u}_f(p_f, h_f)\gamma_\mu\left[\frac{\langle r^2\rangle^{fi}}{6} + a^{fi}\gamma_5\right]u_i(p_i, h_i)J^\mu_{EM}(x). \quad (5)$$

The neutrino initial and final states are given by

$$u_i(\mathbf{p}, h) = \sqrt{E_i + m_i}\begin{pmatrix}\chi^{(h)} \\ \frac{\boldsymbol{\sigma}\mathbf{p}}{E+m_i}\chi^{(h)}\end{pmatrix}, \quad (6)$$

where $\chi^{(h)}$ defines the neutrino helicity state $\chi^{(+)} = (1\ \ 0)^T$ and $\chi^{(-)} = (0\ \ 1)^T$. Substituting (6) into equation (5) we get

$$H_J^{(m)fi} = 2\chi^{(h_f)\dagger}\left\{\boldsymbol{J}_\parallel^{EM}\left(\frac{\langle r^2\rangle^{fi}}{6} + a^{fi}\sigma_3\right) + \boldsymbol{J}_\perp^{EM}\left[\left(\sigma_1\gamma_{fi}^{-1}\cos\xi + \sigma_2\gamma_{fi}^{-1}\sin\xi\right)a^{fi} + \left(i\sigma_1\tilde{\gamma}_{fi}^{-1}\sin\xi - i\sigma_2\tilde{\gamma}_{fi}^{-1}\cos\xi\right)\frac{\langle r^2\rangle^{fi}}{6}\right]\right\}\chi^{(h_i)}, \quad (7)$$

where $\boldsymbol{J}_\parallel^{EM}$ is the longitudinal (in respect to z-axis of the neutrino propagation) electric current and $\boldsymbol{J}_\perp^{EM}$ is the transversal component of the ccurrent, ξ is the angle between the fixed x-axis that is perpendicular to z-axis. The gamma factors are given by

$$\gamma_\alpha^{-1} = \frac{m_\alpha}{E_\alpha}, \quad \gamma_{\alpha\beta}^{-1} = \frac{1}{2}\left(\gamma_\alpha^{-1} + \gamma_\beta^{-1}\right), \quad \tilde{\gamma}_{\alpha\beta}^{-1} = \frac{1}{2}\left(\gamma_\alpha^{-1} - \gamma_\beta^{-1}\right). \quad (8)$$

Now consider contribution of the longitudinal $\boldsymbol{J}_\parallel^{EM}$ and the transversal $\boldsymbol{J}_\perp^{EM}$ components of the electric current separately.

The longitudinal current $J_\parallel^{EM}$ contribution

In the flavour basis $\nu_f = (\nu_e^L, \nu_\alpha^L, \nu_e^R, \nu_\alpha^R)$ for the corresponding part of the interaction Hamiltonian we get

$$H_{J_\parallel}^{(f)} = 2J_\parallel^{EM}\begin{pmatrix} \frac{\langle r^2\rangle^{ee}}{6} - a^{ee} & \frac{\langle r^2\rangle^{e\alpha}}{6} - a^{e\alpha} & 0 & 0 \\ \frac{\langle r^2\rangle^{e\alpha}}{6} - a^{e\alpha} & \frac{\langle r^2\rangle^{\alpha\alpha}}{6} - a^{\alpha} & 0 & 0 \\ 0 & 0 & \frac{\langle r^2\rangle^{ee}}{6} + a^{ee} & \frac{\langle r^2\rangle^{e\alpha}}{6} + a^{e\alpha} \\ 0 & 0 & \frac{\langle r^2\rangle^{e\alpha}}{6} + a^{e\alpha} & \frac{\langle r^2\rangle^{\alpha\alpha}}{6} + a^{\alpha\alpha} \end{pmatrix}, \quad (9)$$

where

$$\begin{aligned} &\langle r^2\rangle^{ee} = \langle r^2\rangle^{11}\cos^2\theta + \langle r^2\rangle^{22}\sin^2\theta + \langle r^2\rangle^{12}\sin 2\theta, \ \ \langle r^2\rangle^{\alpha\alpha} = \langle r^2\rangle^{11}\sin^2\theta + \langle r^2\rangle^{22}\cos^2\theta - \langle r^2\rangle^{12}\sin 2\theta, \\ &a^{ee} = a^{11}\cos^2\theta + a^{22}\sin^2\theta + a^{12}\sin 2\theta, \ \ a^{\alpha\alpha} = a^{11}\sin^2\theta + a^{22}\cos^2\theta - a^{12}\sin 2\theta, \\ &\langle r^2\rangle^{e\alpha} = \langle r^2\rangle^{12}\cos 2\theta + \frac{1}{2}\left(\langle r^2\rangle^{22} - \langle r^2\rangle^{11}\right)\sin 2\theta, \ \ a^{e\alpha} = a^{12}\cos 2\theta + \frac{1}{2}\left(a^{22} - a^{11}\right)\sin 2\theta. \end{aligned} \quad (10)$$

It can be seen that the neutrino interaction with the longitudinal current $\boldsymbol{J}_\parallel^{EM}$ modifies the neutrino flavour oscillations.

The transversal current $J_\perp^{EM}$ contribution

In the flavour basis for the corresponding part of the interaction Hamiltonian we get

$$H_{J_\perp}^{(f)} = 2J_\perp^{EM}\begin{pmatrix} 0 & 0 & \left(\frac{a}{\gamma}\right)_{ee}e^{i\xi} & \left[\tilde{\gamma}_{12}^{-1}\frac{\langle r^2\rangle^{12}}{6} + \left(\frac{a}{\gamma}\right)_{e\alpha}\right]e^{i\xi} \\ 0 & 0 & \left[-\tilde{\gamma}_{12}^{-1}\frac{\langle r^2\rangle^{12}}{6} + \left(\frac{a}{\gamma}\right)_{e\alpha}\right]e^{i\xi} & \left(\frac{a}{\gamma}\right)_{\alpha\alpha}e^{i\xi} \\ \left(\frac{a}{\gamma}\right)_{ee}e^{-i\xi} & \left[-\tilde{\gamma}_{12}^{-1}\frac{\langle r^2\rangle^{12}}{6} + \left(\frac{a}{\gamma}\right)_{e\alpha}\right]e^{-i\xi} & 0 & 0 \\ \left[\tilde{\gamma}_{12}^{-1}\frac{\langle r^2\rangle^{12}}{6} + \left(\frac{a}{\gamma}\right)_{e\alpha}\right]e^{-i\xi} & \left(\frac{a}{\gamma}\right)_{\alpha\alpha}e^{-i\xi} & 0 & 0 \end{pmatrix}, \quad (11)$$

where

$$\begin{aligned} &\left(\frac{a}{\gamma}\right)_{ee} = \frac{a^{11}}{\gamma_{11}}\cos^2\theta + \frac{a^{22}}{\gamma_{22}}\sin^2\theta + \frac{a^{12}}{\gamma_{12}}\sin 2\theta, \ \ \left(\frac{a}{\gamma}\right)_{\alpha\alpha} = \frac{a^{11}}{\gamma_{11}}\sin^2\theta + \frac{a^{22}}{\gamma_{22}}\cos^2\theta - \frac{a^{12}}{\gamma_{12}}\sin 2\theta, \\ &\left(\frac{a}{\gamma}\right)_{e\alpha} = \frac{a^{12}}{\gamma_{12}}\cos 2\theta + \frac{1}{2}\left(\frac{a^{22}}{\gamma_{22}} - \frac{a^{11}}{\gamma_{11}}\right)\sin 2\theta. \end{aligned} \quad (12)$$

From these expressions it follows that the neutrino charge radius and anapole moment interactions with the transversal electric current can generate neutrino spin and spin-flavour oscillations.

Neutrino interaction Hamiltonian in matter and external magnetic field

Neutrino also interacts with moving media and interacts with external magnetic field though neutrino magnetic moment. From weak interaction with moving matter one gets (see [2])

$$H_{mat} = \frac{G_F}{2\sqrt{2}}\begin{pmatrix} 4n_e\left(1 - v_{e||}\right) - 2n_n\left(1 - v_{n||}\right) & 0 & (2n_e v_{e\perp} - n_n v_{n\perp})\left(\frac{\eta}{\gamma}\right)_{ee} & (2n_e v_{e\perp} - n_n v_{n\perp})\left(\frac{\eta}{\gamma}\right)_{e\alpha} \\ 0 & -2n_n\left(1 - v_{n||}\right) & -n_n v_{n\perp}\left(\frac{\eta}{\gamma}\right)_{e\alpha} & -n_n v_{n\perp}\left(\frac{\eta}{\gamma}\right)_{\alpha\alpha} \\ (2n_e v_{e\perp} - n_n v_{n\perp})\left(\frac{\eta}{\gamma}\right)_{ee} & -n_n v_{n\perp}\left(\frac{\eta}{\gamma}\right)_{e\alpha} & 0 & 0 \\ (2n_e v_{e\perp} - n_n v_{n\perp})\left(\frac{\eta}{\gamma}\right)_{e\alpha} & -n_n v_{n\perp}\left(\frac{\eta}{\gamma}\right)_{\alpha\alpha} & 0 & 0 \end{pmatrix}, \quad (13)$$

where n_n and n_e are the neutron and electron density profiles, $\boldsymbol{v}_{n,e} = \boldsymbol{v}_{n,e||} + \boldsymbol{v}_{n,e\perp}$ are the neutrons and electrons velocities and

$$\left(\frac{\eta}{\gamma}\right)_{ee} = \frac{\cos^2\theta}{\gamma_{11}} + \frac{\sin^2\theta}{\gamma_{22}}, \quad \left(\frac{\eta}{\gamma}\right)_{\alpha\alpha} = \frac{\sin^2\theta}{\gamma_{11}} + \frac{\cos^2\theta}{\gamma_{22}}, \quad \left(\frac{\eta}{\gamma}\right)_{e\alpha} = \frac{\sin 2\theta}{\tilde{\gamma}_{21}}. \quad (14)$$

The Hamiltonian that accounts for the neutrino magnetic moment interaction with two components of an external magnetic field $B = B_{||} + B_\perp$ reads

$$H_B = \begin{pmatrix} \left(\frac{\mu}{\gamma}\right)_{ee}B_{||} & \left(\frac{\mu}{\gamma}\right)_{e\alpha}B_{||} & -\mu_{ee}B_\perp e^{i\phi} & -\mu_{e\alpha}B_\perp e^{i\phi} \\ \left(\frac{\mu}{\gamma}\right)_{e\alpha}B_{||} & \left(\frac{\mu}{\gamma}\right)_{\alpha\alpha}B_{||} & -\mu_{e\alpha}B_\perp e^{i\phi} & -\mu_{\alpha\alpha}B_\perp e^{i\phi} \\ -\mu_{ee}B_\perp e^{-i\phi} & -\mu_{e\alpha}B_\perp e^{-i\phi} & -\left(\frac{\mu}{\gamma}\right)_{ee}B_{||} & -\left(\frac{\mu}{\gamma}\right)_{e\alpha}B_{||} \\ -\mu_{e\alpha}B_\perp e^{-i\phi} & -\mu_{\alpha\alpha}B_\perp e^{-i\phi} & -\left(\frac{\mu}{\gamma}\right)_{e\alpha}B_{||} & -\left(\frac{\mu}{\gamma}\right)_{\alpha\alpha}B_{||} \end{pmatrix}, \quad (15)$$

where ϕ is the angle between $\boldsymbol{v}_\perp$ and $\boldsymbol{B}_\perp$. The components of the effective magnetic moment in the flavour basis are expressed through the components in the mass basis

$$\begin{aligned} &\mu_{ee} = \mu_{11}\cos^2\theta + \mu_{22}\sin^2\theta + \mu_{12}\sin 2\theta, \ \ \mu_{e\alpha} = \mu_{12}\cos 2\theta + \frac{1}{2}\left(\mu_{22} - \mu_{11}\right)\sin 2\theta, \\ &\mu_{\alpha\alpha} = \mu_{11}\sin^2\theta + \mu_{22}\cos^2\theta - \mu_{12}\sin 2\theta, \end{aligned} \quad (16)$$

and

$$\begin{aligned} &\left(\frac{\mu}{\gamma}\right)_{ee} = \frac{\mu_{11}}{\gamma_{11}}\cos^2\theta + \frac{\mu_{22}}{\gamma_{22}}\sin^2\theta + \frac{\mu_{12}}{\gamma_{12}}\sin 2\theta, \ \ \left(\frac{\mu}{\gamma}\right)_{e\alpha} = \frac{\mu_{12}}{\gamma_{12}}\cos 2\theta + \frac{1}{2}\left(\frac{\mu_{22}}{\gamma_{22}} - \frac{\mu_{11}}{\gamma_{11}}\right)\sin 2\theta, \\ &\left(\frac{\mu}{\gamma}\right)_{\alpha\alpha} = \frac{\mu_{11}}{\gamma_{11}}\sin^2\theta + \frac{\mu_{22}}{\gamma_{22}}\cos^2\theta - \frac{\mu_{12}}{\gamma_{12}}\sin 2\theta. \end{aligned} \quad (17)$$

In derivations of the neutrino oscillations probabilities we take into account all interaction Hamiltonians and also the vacuum Hamiltonian.

Neutrino spin oscillations $\nu_e^L \leftrightarrow \nu_e^R$

Consider two neutrino states with different helicities: (ν_e^L, ν_e^R). The corresponding neutrino oscillations are governed by the evolution equation

$$i\frac{d}{dt}\begin{pmatrix}\nu_e^L \\ \nu_e^R\end{pmatrix} = \left[\frac{G_F}{2\sqrt{2}}\begin{pmatrix} 4n_e\left(1 - v_{e||}\right) - 2n_n\left(1 - v_{n||}\right) & (2n_e v_{e\perp} - n_n v_{n\perp})\left(\frac{\eta}{\gamma}\right)_{ee} \\ (2n_e v_{e\perp} - n_n v_{n\perp})\left(\frac{\eta}{\gamma}\right)_{ee} & 0 \end{pmatrix} + \begin{pmatrix} \left(\frac{\mu}{\gamma}\right)_{ee}B_{||} & -\mu_{ee}B_\perp e^{i\phi} \\ -\mu_{ee}B_\perp e^{-i\phi} & -\left(\frac{\mu}{\gamma}\right)_{ee}B_{||} \end{pmatrix} + \begin{pmatrix} -2J_\parallel^{EM}a^{ee} & 2J_\perp^{EM}\left(\frac{a}{\gamma}\right)_{ee}e^{i\chi} \\ 2J_\perp^{EM}\left(\frac{a}{\gamma}\right)_{ee}e^{-i\chi} & 2J_\parallel^{EM}a^{ee} \end{pmatrix}\right]\begin{pmatrix}\nu_e^L \\ \nu_e^R\end{pmatrix}. \quad (18)$$

For the oscillation $\nu_e^L \leftrightarrow \nu_e^R$ probability we get

$$P_{\nu_e^L\to\nu_e^R}(x) = \sin^2 2\theta_{\text{eff}}\sin^2\frac{\pi x}{L_{\text{eff}}}, \quad \sin^2 2\theta_{\text{eff}} = \frac{E^2_{\text{eff}}}{E^2_{\text{eff}} + \Delta^2_{\text{eff}}}, \quad L_{\text{eff}} = \frac{\pi}{\sqrt{E^2_{\text{eff}} + \Delta^2_{\text{eff}}}}, \quad (19)$$

where E^2_{eff} and Δ^2_{eff} are expressed in terms of the elements H_{ij} of the Hamiltonian:

$$E^2_{eff} = 4|H_{12}|^2 = 4\left[\frac{G_F}{2\sqrt{2}}(2n_e v_{e\perp} - n_n v_{n\perp})\left(\frac{\eta}{\gamma}\right)_{ee} - \mu_{ee}B_\perp\cos\phi + 2J_\perp^{EM}\left(\frac{a}{\gamma}\right)_{ee}\cos\xi\right]^2 + 4\left[\mu_{ee}B_\perp\sin\phi - 2J_\perp^{EM}\left(\frac{a}{\gamma}\right)_{ee}\sin\xi\right]^2, \quad (20)$$

$$\Delta^2_{eff} = (H_{11} - H_{22})^2 = \left[\frac{G_F}{\sqrt{2}}\left(2n_e\left(1 - v_{e||}\right) - n_n\left(1 - v_{n||}\right)\right) + 2\left(\frac{\mu}{\gamma}\right)_{ee}B_{||} - 4J_\parallel^{EM}a^{ee}\right]^2. \quad (21)$$

It follows that whereas the spin oscillations can be generated by the neutrino anapole moment interactions with an external electric current, the interaction due to the charge radius does not produce the spin oscillations. Thus, these peculiarities can be used for disintegration of the anapole moment and charge radius effects in neutrino interactions.

Neutrino spin-flavour oscillations $\nu_e^L \leftrightarrow \nu_\alpha^R$

Now consider two neutrino flavour states with two different helicities: (ν_e^L, ν_α^R). The corresponding neutrino oscillations are governed by the evolution equation

$$i\frac{d}{dt}\begin{pmatrix}\nu_e^L \\ \nu_\alpha^R\end{pmatrix} = \left[\frac{\Delta m^2}{4E}\begin{pmatrix} -\cos 2\theta & 0 \\ 0 & \cos 2\theta \end{pmatrix} + \frac{G_F}{2\sqrt{2}}\begin{pmatrix} 4n_e\left(1 - v_{e||}\right) - 2n_n\left(1 - v_{n||}\right) & (2n_e v_{e\perp} - n_n v_{n\perp})\left(\frac{\eta}{\gamma}\right)_{e\alpha} \\ (2n_e v_{e\perp} - n_n v_{n\perp})\left(\frac{\eta}{\gamma}\right)_{e\alpha} & 0 \end{pmatrix} + \begin{pmatrix} \left(\frac{\mu}{\gamma}\right)_{ee}B_{||} & -\mu_{e\alpha}B_\perp e^{i\phi} \\ -\mu_{e\alpha}B_\perp e^{-i\phi} & -\left(\frac{\mu}{\gamma}\right)_{\alpha\alpha}B_{||} \end{pmatrix} + \begin{pmatrix} 2J_\parallel^{EM}\left(\frac{\langle r^2\rangle^{ee}}{6} - a^{ee}\right) & 2J_\perp^{EM}\left[\tilde{\gamma}_{12}^{-1}\frac{\langle r^2\rangle^{12}}{6} + \left(\frac{a}{\gamma}\right)_{e\alpha}\right]e^{i\xi} \\ 2J_\perp^{EM}\left[\tilde{\gamma}_{12}^{-1}\frac{\langle r^2\rangle^{12}}{6} + \left(\frac{a}{\gamma}\right)_{e\alpha}\right]e^{-i\xi} & 2J_\parallel^{EM}\left(\frac{\langle r^2\rangle^{\alpha\alpha}}{6} + a^{\alpha\alpha}\right) \end{pmatrix}\right]\begin{pmatrix}\nu_e^L \\ \nu_\alpha^R\end{pmatrix}. \quad (22)$$

For the case of the neutrino oscillations $\nu_e^L \leftrightarrow \nu_\alpha^R$ the probability is again given by (19) with

$$E^2_{eff} = 4\left[\frac{G_F}{2\sqrt{2}}(2n_e v_{e\perp} - n_n v_{n\perp})\left(\frac{\eta}{\gamma}\right)_{e\alpha} - \mu_{e\alpha}B_\perp\cos\phi + 2J_\perp^{EM}\left[\tilde{\gamma}_{12}^{-1}\frac{\langle r^2\rangle^{12}}{6} + \left(\frac{a}{\gamma}\right)_{e\alpha}\right]\cos\xi\right]^2 + 4\left[\mu_{e\alpha}B_\perp\sin\phi - J_\perp^{EM}\left[\tilde{\gamma}_{12}^{-1}\frac{\langle r^2\rangle^{12}}{6} + \left(\frac{a}{\gamma}\right)_{e\alpha}\right]\sin\xi\right]^2, \quad (23)$$

$$\Delta^2_{eff} = \left[-\frac{\Delta m^2}{2E}\cos\theta + \frac{G_F}{\sqrt{2}}\left(2n_e\left(1 - v_{e||}\right) - n_n\left(1 - v_{n||}\right)\right) + \left[\left(\frac{\mu}{\gamma}\right)_{ee} + \left(\frac{\mu}{\gamma}\right)_{\alpha\alpha}\right]B_{||} + J_\parallel^{EM}\left(\frac{\langle r^2\rangle^{ee} - \langle r^2\rangle^{\alpha\alpha}}{6} - a^{ee} + a^{\alpha\alpha}\right)\right]^2. \quad (24)$$

Contrary to the previous case, these expressions depend on both the neutrino anapole moment and the charge radius.

Acknowledgments

The work is supported by the Russian Science Foundation under grant No.22-22-00384.

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