Wave packet treatment of neutrino flavour and spin oscillations in galactic and extragalactic magnetic fields

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High-energy neutrinos point sources

- Recent data analyses present evidence of observation of astrophysical neutrinos emanating from distant objects, such as active galactic nuclei and blazars:

1. IceCube Collaboration, *"Evidence for neutrino emission from the nearby active galaxy NGC 1068"*, Science 378 (2022) 6619, 538-543,
2. IceCube Collaboration, *"TXS 0506+056 with Updated IceCube Data"*, PoS ICRC2023 (2023) 1465,
3. Baikal-GVD Collaboration, *"Baikal-GVD Astrophysical Neutrino Candidate near the Blazar TXS~0506+056"*, PoS ICRC2023 1457.

- Neutrinos are unique astrophysical messengers, since unlike charges particles they are not deflected by magnetic field. However, they interact with a magnetic field via magnetic moments.

High-energy neutrinos flavour ratios

- Standard neutrino oscillations in vacuum predict the following flavour ratios at the terrestrial neutrino telescope:

$$r_\alpha = \sum_\beta r^0_\beta \sum_i |U_{\alpha i}|^2 |U_{\beta i}|^2$$

where r^0_β are **flavour ratios at the neutrino source** $(\alpha, \beta = e, \mu, \tau)$.

- Pion decay neutrino production: $r^0 = \left(\frac{1}{2}, \frac{2}{3}, 0\right)$ and $r \approx \left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}\right)$.

$$\pi^+ \to \mu^+ + \nu_\mu,$$
$$\mu^+ \to e^+ + \nu_e + \bar{\nu}_\mu$$

⇒ $\nu_e : \nu_\mu : \nu_\tau$ (1:2:0)

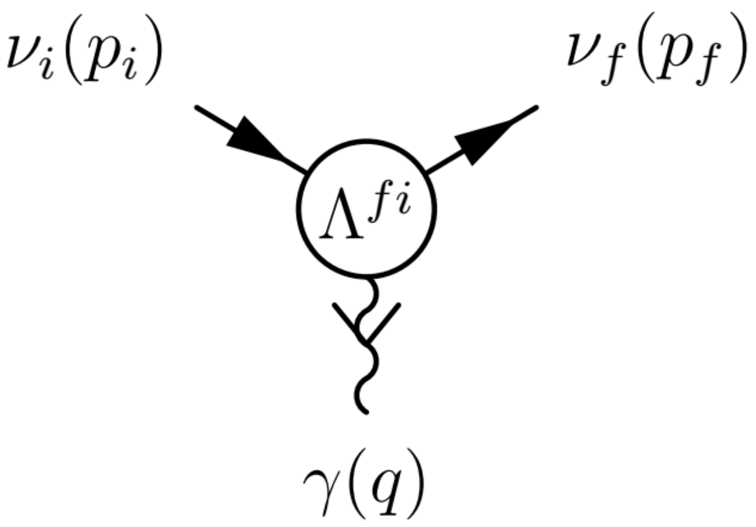
M.Bustamante, J.Beacom, W.Winter, *"Theoretically palatable flavor combinations of astrophysical neutrinos"*, Phys.Rev.Lett. 115 (2015) 16

Flavour ratios as a probe of BSM physics

- **Quantum gravity**
  IceCube Collaboration, "*Searching for Decoherence from Quantum Gravity at the IceCube South Pole Neutrino Observatory*", arXiv 2308.00105
- **Lorentz violation**
  D.Hooper, D.Morgan, E.Winstanley, "*Lorentz and CPT Invariance Violation In High-Energy Neutrinos*", Phys.Rev.D 72 (2005) 065009
- **Neutrino decay**
  P.Baerwald, M.Bustamante, W.Winter, "*Neutrino Decays over Cosmological Distances and the Implications for Neutrino Telescopes*", JCAP 10 (2012) 020
- **Sterile neutrinos**
  A.Esmailia , Y.Farzan, "*Implications of the Pseudo-Dirac Scenario for Ultra High Energy Neutrinos from GRBs*", JCAP 12 (2012) 014

In this talk we report possible effects of neutrino interaction with a magnetic field on flavour ratios

Neutrino electromagnetic properties



$$\mathcal{H}_{\mathrm{em}}^{(\nu)}(x) = j_{\mu}^{(\nu)}(x) A^{\mu}(x) = \sum_{k,j=1}^{N} \overline{\nu_k}(x) \Lambda_{\mu}^{kj} \nu_j(x) A^{\mu}(x),$$

The vertex function is parametrized in terms of **charge, anapole, electric and magnetic form factors**:

$$\Lambda_{\mu}(q) = \left(\gamma_{\mu} - q_{\mu} \not{q}/q^2\right) \left[\mathbb{f}_Q(q^2) + \mathbb{f}_A(q^2) q^2 \gamma_5\right] - i\sigma_{\mu\nu} q^{\nu} \left[\mathbb{f}_M(q^2) + i\mathbb{f}_E(q^2)\gamma_5\right]$$

$\mathbb{f}_M^{fi}(0) = \mu_{fi}$ - neutrino magnetic moments

C.Giunti, A.Studenikin, *"Neutrino electromagnetic interactions: A window to new physics"*, Rev.Mod.Phys. 87 (2015) 531

Neutrino magnetic moments matrix

CPT-invariance + hermicity:

- Magnetic moments matrix for **Dirac** neutrinos is **real and symmetric**:

$$\mu^D = \begin{pmatrix} \mu_{11} & \mu_{12} & \mu_{13} \\ \mu_{12} & \mu_{22} & \mu_{23} \\ \mu_{13} & \mu_{23} & \mu_{33} \end{pmatrix}$$

- Magnetic moments matrix for **Majorana** neutrinos is **imaginary and asymmetric**:

$$\mu^M = \begin{pmatrix} 0 & i\mu_{12} & i\mu_{13} \\ -i\mu_{12} & 0 & i\mu_{23} \\ -i\mu_{13} & -i\mu_{23} & 0 \end{pmatrix}$$

- Thus, Dirac and Majorana neutrinos can be distinguished by their **electromagnetic properties**
  A.Popov, A.Studenikin, *"Manifestations of nonzero Majorana CP-violating phases in oscillations of supernova neutrinos"*, Phys.Rev.D 103 (2021) 11, 115027

Neutrino magnetic moments

Theory (Standard Model):

$$\mu_{ii}^{D} = \frac{3eG_F m_i}{8\sqrt{2}\pi^2} \approx 3.2 \times 10^{-19}\left(\frac{m_i}{1\ eV}\right)\mu_B$$

K.Fujikawa, R.Shrock, *"The Magnetic Moment of a Massive Neutrino and Neutrino Spin Rotation"*, Phys.Rev.Lett. 45 (1980) 963

Experiment:

$$\mu_\nu < 6.4 \times 10^{-12}\ \mu_{\mathrm{B}}$$

E.Aprile *et al.* [XENON collaboration], *"Search for New Physics in Electronic Recoil Data from XENONnT"*, Phys.Rev.Lett. 129 (2022) 16, 161805

Upper bounds from astrophysical neutrinos:

R.L. Workman et al. (Particle Data Group), Prog. Theor. Exp. Phys. 2022, 083C01 (2022)

$$\mu_\nu \lesssim 10^{-12}\mu_B$$

Neutrino evolution in a magnetic field

- Neutrino evolution in a magnetic field is described by the following Dirac equation:

$$(i\gamma^{\mu}\partial_{\mu} - m_i)\nu_i(x) - \sum_{k} \mu_{ik}\boldsymbol{\Sigma}\boldsymbol{B}\nu_k(x) = 0, \qquad \textbf{(1)}$$

A.Popov, A.Studenikin, *"Manifestations of nonzero Majorana CP-violating phases in oscillations of supernova neutrinos"*, Phys.Rev.D 103 (2021) 11, 115027

- For the case of wave packet description of neutrino oscillations, and neglecting transition magnetic moments, Equation (1) can be rewritten as

$$i\partial_t\nu_i(p,t) = [m_i\gamma_0 + \gamma_0\gamma_1 p]\nu_i(p,t) + \mu_i\gamma_0\boldsymbol{\Sigma}\boldsymbol{B}(\langle x_i(t)\rangle)\nu_i(p,t) = 0 \qquad \textbf{(2)}$$

Here $\langle x_i(t)\rangle$ are expectations of massive neutrino states wavepackets coordinates.

We solve **Equation (2):**

1. Analytically for the case of uniform magnetic field,
2. Numerically for realistic galactic magnetic field model.

Analytical solution

- We assume that neutrino wave function is described by a Gaussian wave packet:

$$\nu_i(p,0) \sim \exp\left(\frac{(p-p_0)^2}{4\sigma_p^2}\right)$$

where σ_p neutrino momentum uncertainty and p_0 is average neutrino momentum.

- The probability of flavour conversion is:

$$P_{\nu_\alpha\to\nu_\beta}(L) = \frac{1}{4}\sum_{i,j}\sum_{s,\sigma} U^*_{\beta i}U_{\alpha i}U_{\beta j}U^*_{\alpha j}\exp\left(-i2\pi\frac{L}{L^{ijs\sigma}_{osc}}\right)\exp\left(-\frac{L^2}{(L^{ijs\sigma}_{coh})^2}\right),$$

where L_{osc} are oscillations lengths and L_{coh} are *coherence lengths*, $i,j = 1,2,3$ and $s,\sigma = \pm 1$.

$$L^{ijss}_{osc} = \frac{4\pi p}{\Delta m^2_{ij}} \text{ and } L^{ii-+}_{osc} = \frac{\pi}{\mu_i B_\perp}$$

- Oscillations probability is a combination of oscillations on (1) vacuum frequencies $\omega^{vac}_{ik} = \frac{\Delta m^2_{ik}}{4p}$ and (2) magnetic frequencies $\omega^B_i = \mu_i B_\perp$.

(see A.Popov, A. Studenikin, *Neutrino eigenstates and flavour, spin and spin-flavour oscillations in a constant magnetic field*, Eur.Phys.J.C 79 (2019) 2, 144 and references therein)

Coherence lengths

For oscillations on vacuum frequencies ω_{ik}^{vac}:

$$L_{coh}^{ijss} \approx \frac{4\sqrt{2}\sigma_x p^2}{\Delta m_{ij}^2},$$

For oscillations on magnetic frequencies ω_i^B:

$$L_{coh}^{ii-+} \approx \frac{\sigma_x p^3}{\mu_i B m_i^2}.$$

where $\sigma_x = 1/2\sigma_p$ is wave packet width in the coordinate space.

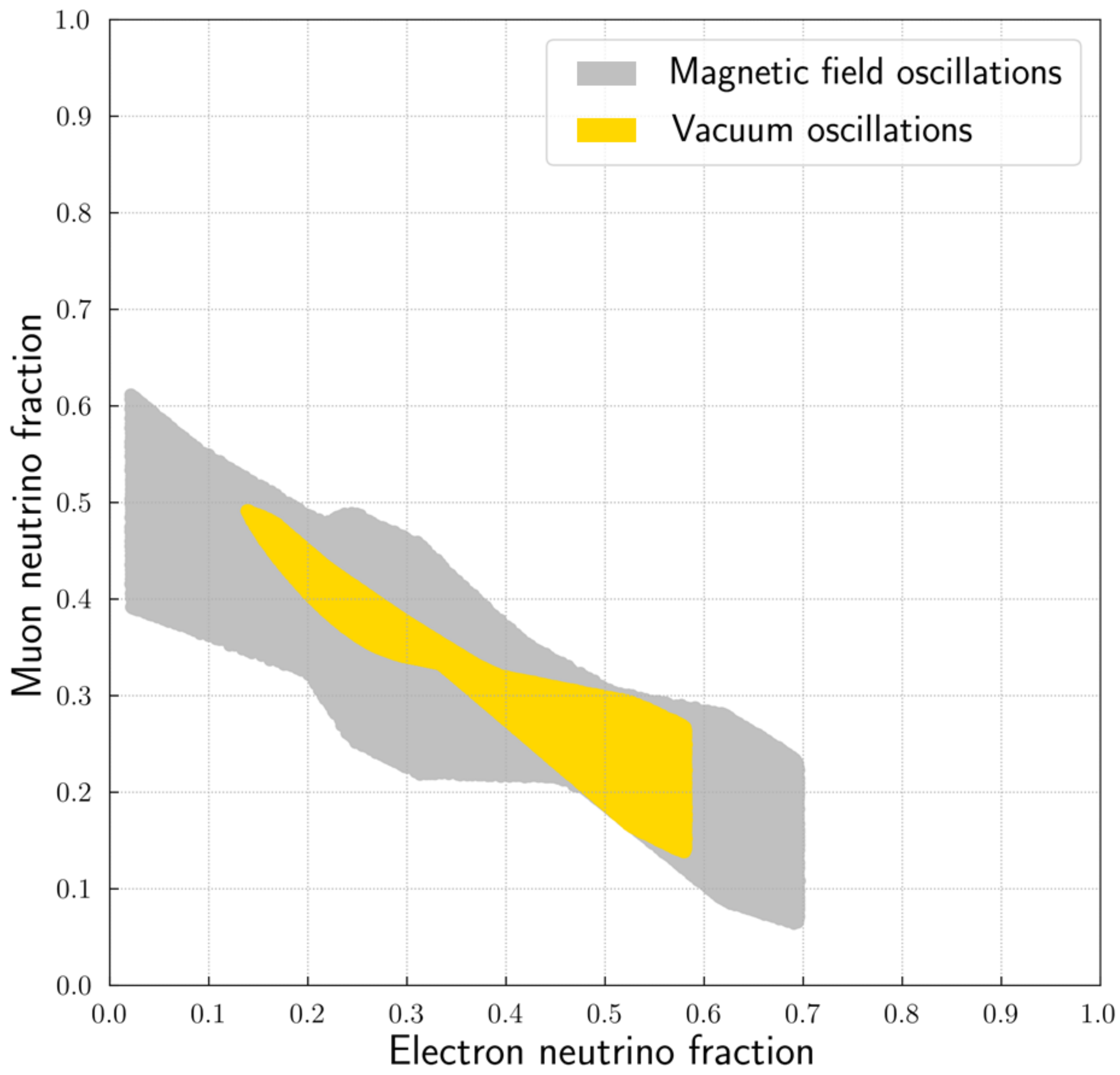
$\sigma_x \sim 10^{-17} \div 10^{-9}$ km for various neutrino creation mechanisms.

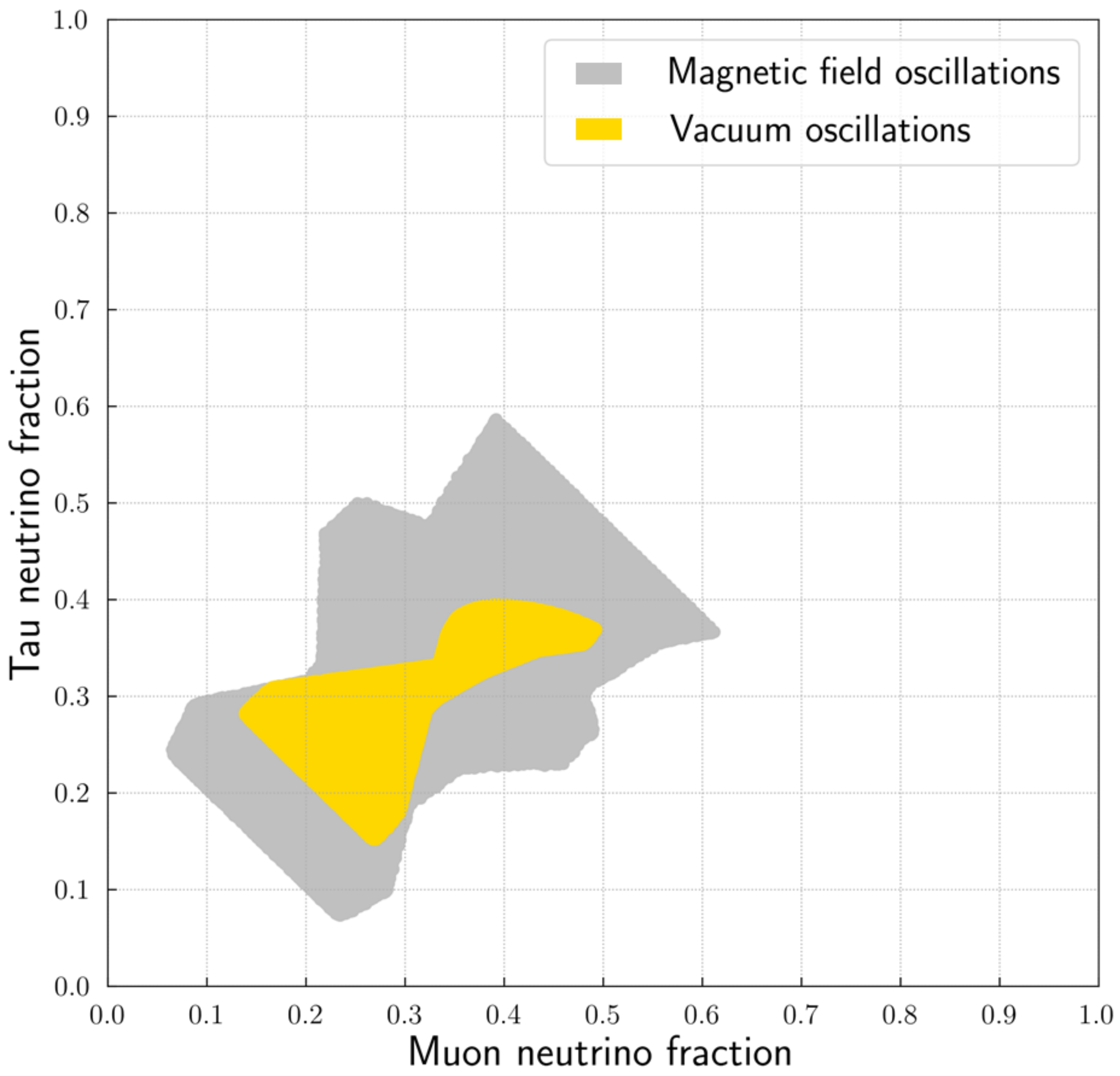
- Thus, oscillations on the vacuum frequencies $\omega_{ik}^{vac} = \frac{\Delta m_{ik}^2}{4p}$ may fade away for the case of astrophysical neutrinos propagation ($L_{coh} \sim 1\ kpc$).
- Oscillations on the magnetic frequencies $\omega_i^B = \mu_i B_\perp$ persist even on astrophysical scale ($L_{coh} \gg 1\ kpc$).

Neutrino oscillations in a Galactic magnetic field

- We use the Galactic magnetic field model provided by R.Jansson, G.Farrar, *"A New Model of the Galactic Magnetic Field"*, Astrophys.J. 757 (2012) 14. The field is of order of $O(\mu G)$.
- We consider high-energy neutrinos originating from Galactic center (see IceCube Collaboration, *"Search for Neutrino Emission at the Galactic Center Region with IceCube"*, PoS ICRC2023 (2023) 1051, and S.Celli, A.Palladino, F.Vissani, *"Neutrinos and γ-rays from the Galactic Center Region After H.E.S.S. Multi-TeV Measurements"*, Eur.Phys.J.C 77 (2017) 2, 66).
- Possible flavour ratios are calculated for different values of neutrino magnetic moments μ_1, μ_2 and μ_3 from $(10^{-13}, 6.4 \cdot 10^{-12})$ Bohr magneton range.
- The obtained flavour ratios are compared to ones predicted by standard vacuum neutrino oscillations.

Predicted flavour ratios



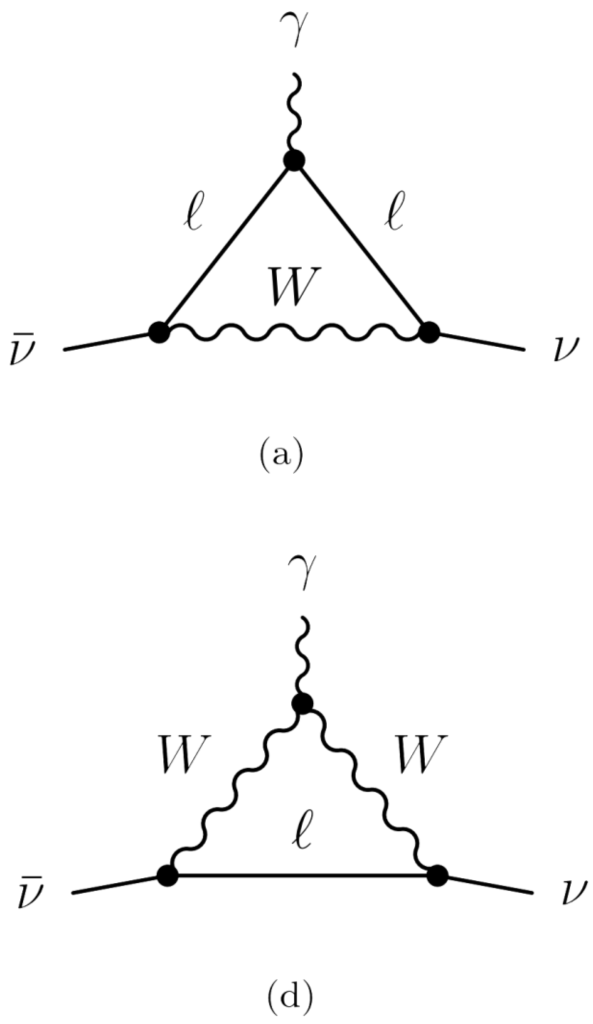


Conclusions

- Neutrino oscillations in a magnetic field are considered accounting for decoherence effects due to wave packets separation.
- The expressions for coherence length are obtained for oscillations on vacuum frequencies and magnetic frequencies. It is shown that the latter is proportional to E_ν^3.
- Possible flavour ratios of neutrinos originating from the Galactic center are obtained. They significantly differ from the vacuum ones for neutrino magnetic moments $\sim 10^{-13} \mu_B$ and higher.
- For the case of Majorana neutrinos, no significant effects were found.

Backup

Neutrino magnetic moment



M.Dvornikov, A.Studenikin, "*Electric charge and magnetic moment of massive neutrino*", Phys.Rev.D. (2004)

Majorana neutrinos

Dirac fermion

$$\Psi_D = \Psi_L + \Psi_R$$

Majorana fermion

$$\Psi_R = \Psi_L^c$$

A Majorana field can be written as $\Psi_M = \Psi_L + \Psi_L^c$

$\Psi_M^c = \Psi_M$ is satisfied for a Majorana field

Majorana mass term violates total lepton number by 2

$$m_i \overline{\nu_i} \nu_i = m_i \overline{(\nu_i^L)^c} \nu_i^L + m_i \overline{\nu_i^L} (\nu_i^L)^c$$

Neutrinos in astrophysics

Known types:

- Solar neutrinos
- Supernova neutrinos
- High-energy neutrinos

Hypothetical sources:

- Diffuse Supernova Neutrino Background
- Gamma-ray bursts
- Active Galactic Nuclei
- Pulsars, magnetars
- Cosmogenic neutrinos
- Relic neutrinos

Oscillations lengths

$$L_{osc}^{ii-+} = \frac{\pi}{\mu_i B_\perp} = 2.17 \cdot \left(\frac{B}{1\ \mu\text{G}}\right)^{-1} \left(\frac{\mu_i}{10^{-11}\mu_B}\right)^{-1} 10^3\ \text{pc},$$

$$L_{osc}^{ijss} = \frac{4\pi p}{\Delta m_{ij}^2} = 5.02 \cdot \left(\frac{\Delta m_{ij}^2}{10^{-5}\ \text{eV}^2}\right)^{-1} \left(\frac{p}{1\ \text{PeV}}\right) \cdot 10^{-2}\ \text{pc}.$$

I.Esteban, M.C.Gonzalez-Garcia, M.Maltoni, T.Schwetz, A.Zhou , *"The fate of hints: updated global analysis of three-flavor neutrino oscillations"*, JHEP 09 (2020) 178; NuFIT 5.2 (2022), www.nu-fit.org

NuFIT 5.2 (2022)

		Normal Ordering (best fit)		Inverted Ordering ($\Delta\chi^2 = 2.3$)	
		bfp $\pm 1\sigma$	3σ range	bfp $\pm 1\sigma$	3σ range
without SK atmospheric data	$\sin^2\theta_{12}$	$0.303^{+0.012}_{-0.011}$	$0.270 \to 0.341$	$0.303^{+0.012}_{-0.011}$	$0.270 \to 0.341$
	$\theta_{12}/^\circ$	$33.41^{+0.75}_{-0.72}$	$31.31 \to 35.74$	$33.41^{+0.75}_{-0.72}$	$31.31 \to 35.74$
	$\sin^2\theta_{23}$	$0.572^{+0.018}_{-0.023}$	$0.406 \to 0.620$	$0.578^{+0.016}_{-0.021}$	$0.412 \to 0.623$
	$\theta_{23}/^\circ$	$49.1^{+1.0}_{-1.3}$	$39.6 \to 51.9$	$49.5^{+0.9}_{-1.2}$	$39.9 \to 52.1$
	$\sin^2\theta_{13}$	$0.02203^{+0.00056}_{-0.00059}$	$0.02029 \to 0.02391$	$0.02219^{+0.00060}_{-0.00057}$	$0.02047 \to 0.02396$
	$\theta_{13}/^\circ$	$8.54^{+0.11}_{-0.12}$	$8.19 \to 8.89$	$8.57^{+0.12}_{-0.11}$	$8.23 \to 8.90$
	$\delta_{CP}/^\circ$	197^{+42}_{-25}	$108 \to 404$	286^{+27}_{-32}	$192 \to 360$
	$\frac{\Delta m^2_{21}}{10^{-5}\ eV^2}$	$7.41^{+0.21}_{-0.20}$	$6.82 \to 8.03$	$7.41^{+0.21}_{-0.20}$	$6.82 \to 8.03$
	$\frac{\Delta m^2_{3\ell}}{10^{-3}\ eV^2}$	$+2.511^{+0.028}_{-0.027}$	$+2.428 \to +2.597$	$-2.498^{+0.032}_{-0.025}$	$-2.581 \to -2.408$