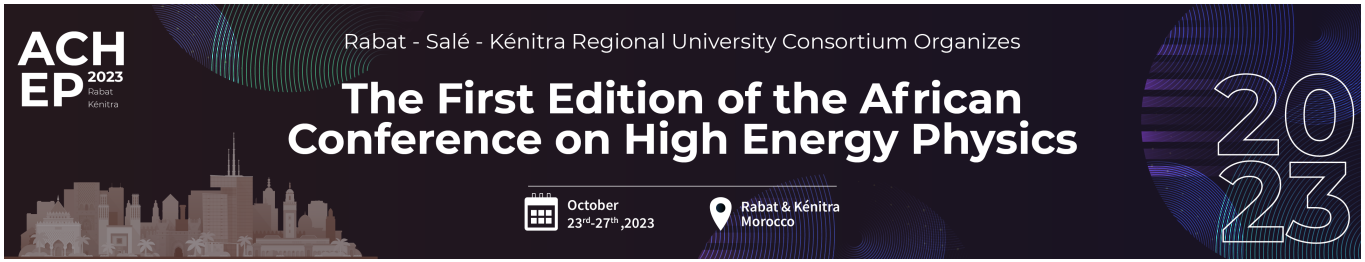

Dynamical and Topological Descriptions of Born-Infeld-AdS Black Hole Thermodynamics

Jamal KHALLOUFI and Hasan EL MOUMNI

Laboratory of Theoretical and High Energy Physics
Physics Department, Faculty of Sciences, Ibn Zohr University, Agadir

23^{th} Octobre 2023

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2. Black Hole as a Thermodynamic System
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4. Conclusion

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- An asymptotically Anti-de-Sitter (AdS) black hole in 4-dimensions is a solution of Einstein equation

$$R_{\mu\nu} + \frac{1}{2} g_{\mu\nu} + \Lambda g_{\mu\nu} = T_{\mu\nu}, \tag{1}$$

where Λ is the cosmological constant parametrized by the AdS radius l according to

$$\Lambda = -\frac{3}{l^2} < 0, \tag{2}$$

and $T_{\mu\nu}$ is the stress-energy tensor.

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- In the vacuum ($T_{\mu\nu} = 0$), the metric of a static spherically symmetric black holes reads

$$ds^2 = -f(r)dt^2 + \frac{dr^2}{f(r)} + r^2 d\Omega_2^2. \tag{3}$$

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- The black hole horizon is located at r_h such that

$$f(r_h) = 0. \tag{4}$$

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Thermodynamics correspondence with black hole :

Thermodynamics		Black hole	
Enthalpy	$H = U + PV$	Mass	M
Temperature	T	Surface gravity	$\frac{\kappa}{2\pi}$
Entropy	S	Horizon area	$\frac{\mathcal{A}}{4}$
Pressure	P	Cosmological constant	$-\frac{\Lambda}{8\pi}$

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- The Lagrangian of Born-Infeld nonlinear electrodynamics is :

$$\mathcal{L}_{BI} = 4b^2 \left(1 - \sqrt{1 + \frac{F_{\mu\nu}F^{\mu\nu}}{2b^2}} \right), \tag{6}$$

the constant b is the Born-Infeld parameter with the dimension of mass that relates to the string tension α' as $b = 1/(2\pi\alpha')$.

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- The metric function $f(r)$ is given by

$$f(r) = 1 + \frac{r^2}{l^2} - \frac{8\pi M}{r} + \frac{2b^2 r^2}{3} \left(1 - \sqrt{1 + \frac{16\pi^2 Q^2}{b^2 r^2}} \right) + \frac{64\pi^2 Q^2}{3r^2} {}_2\mathcal{F}_1 \left[\frac{1}{4}, \frac{1}{2}, \frac{5}{4}, -\frac{16\pi^2 Q^2}{b^2 r^2} \right]. \tag{7}$$

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- Using $f(r)$ function, we can compute all thermodynamic variables $(M, T, S, V, \Phi, ...)$ characterizing the black hole

- In order to study different stable phases and the thermodynamic transitions between them, we define the free energy :

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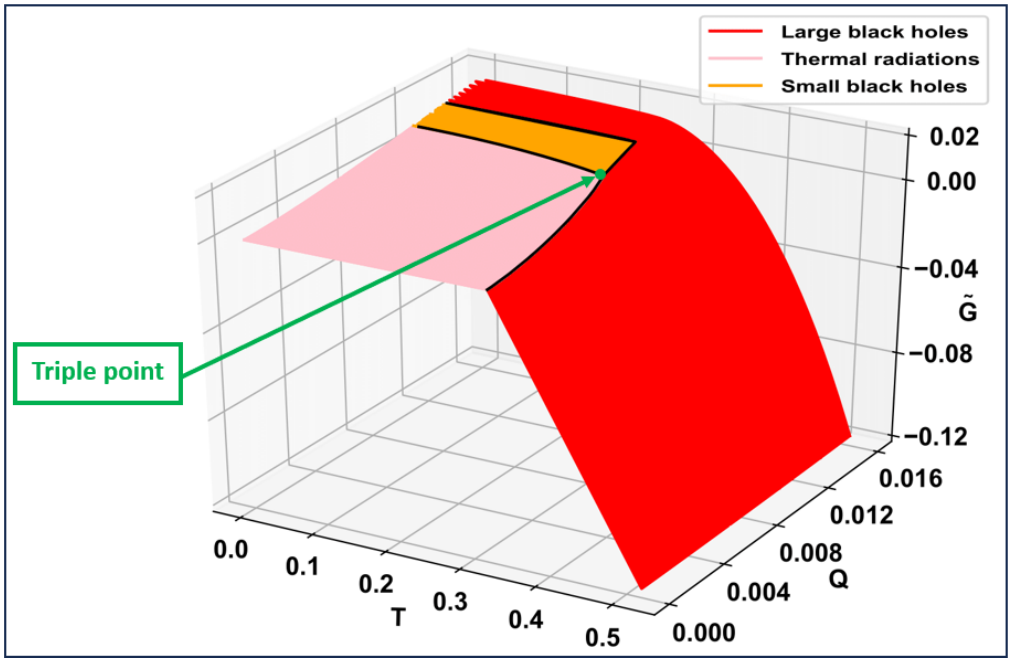

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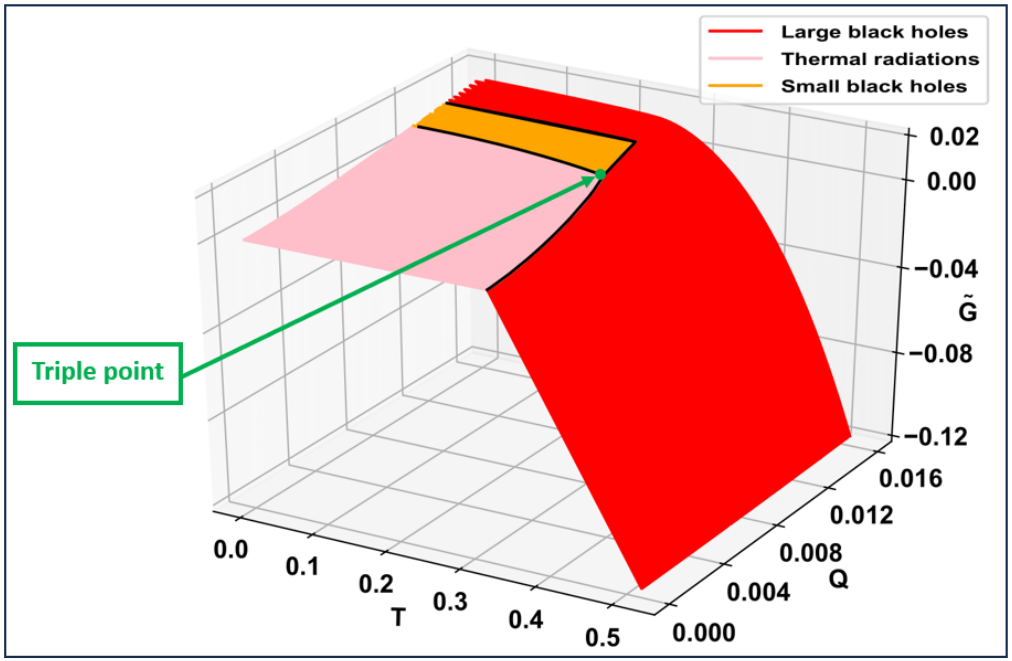

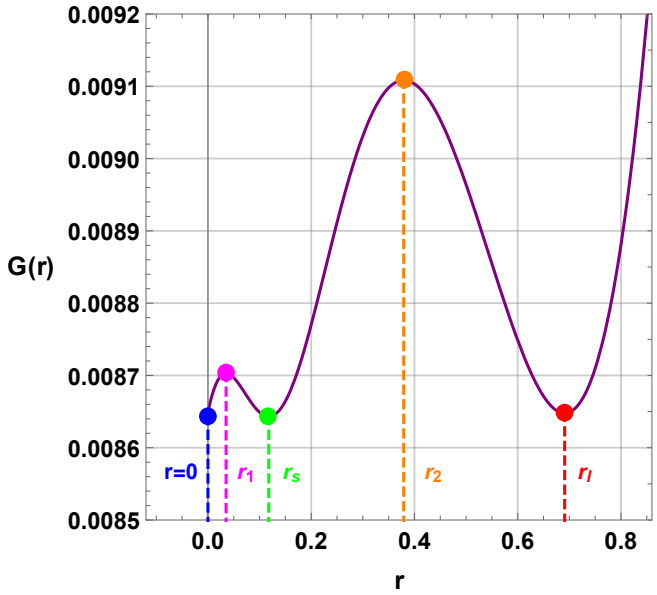

The Fokker-Planck equation for the probabilistic evolution on the free energy landscape is explicitly given by

$$\frac{\partial \rho(r,t)}{\partial t} = D\frac{\partial}{\partial r}\left[e^{-\beta G(r)}\frac{\partial}{\partial r}\left[e^{\beta G(r)}\rho(r,t)\right]\right], \tag{9}$$

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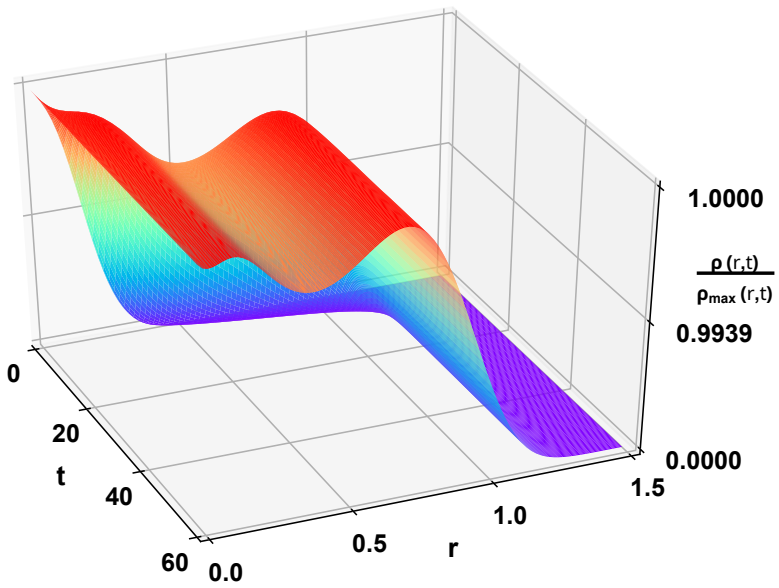
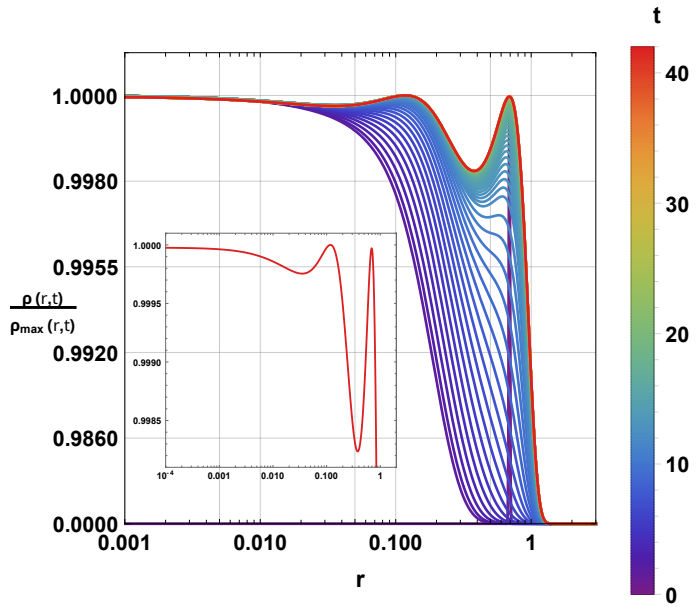
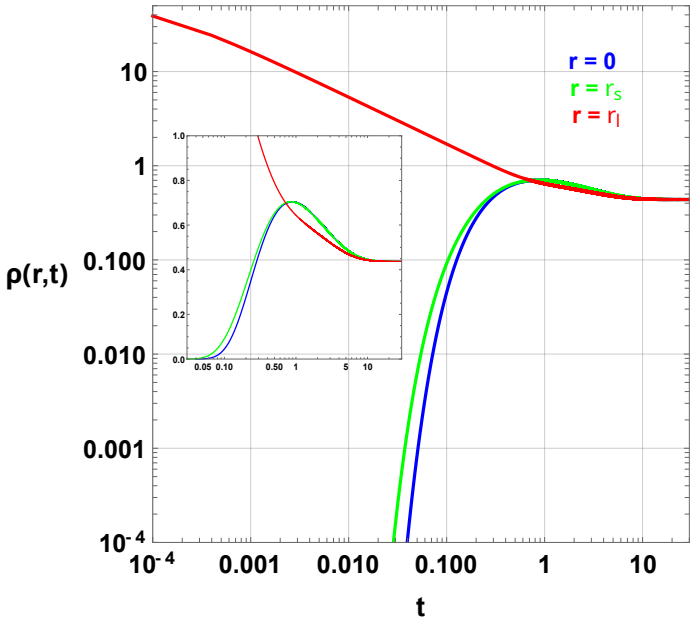
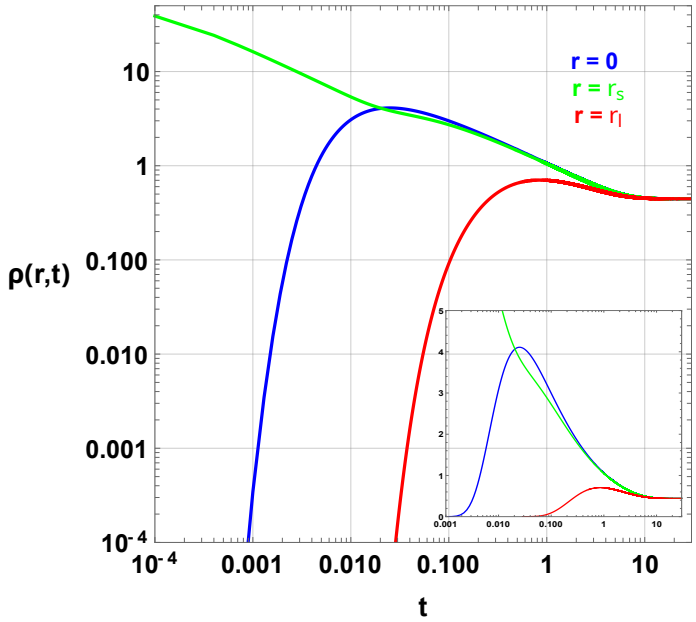




- Behaviors of the probability $\rho(r,t)$ at the triple point, when the initial Gaussian wave packet is peaked at the large and small black hole states





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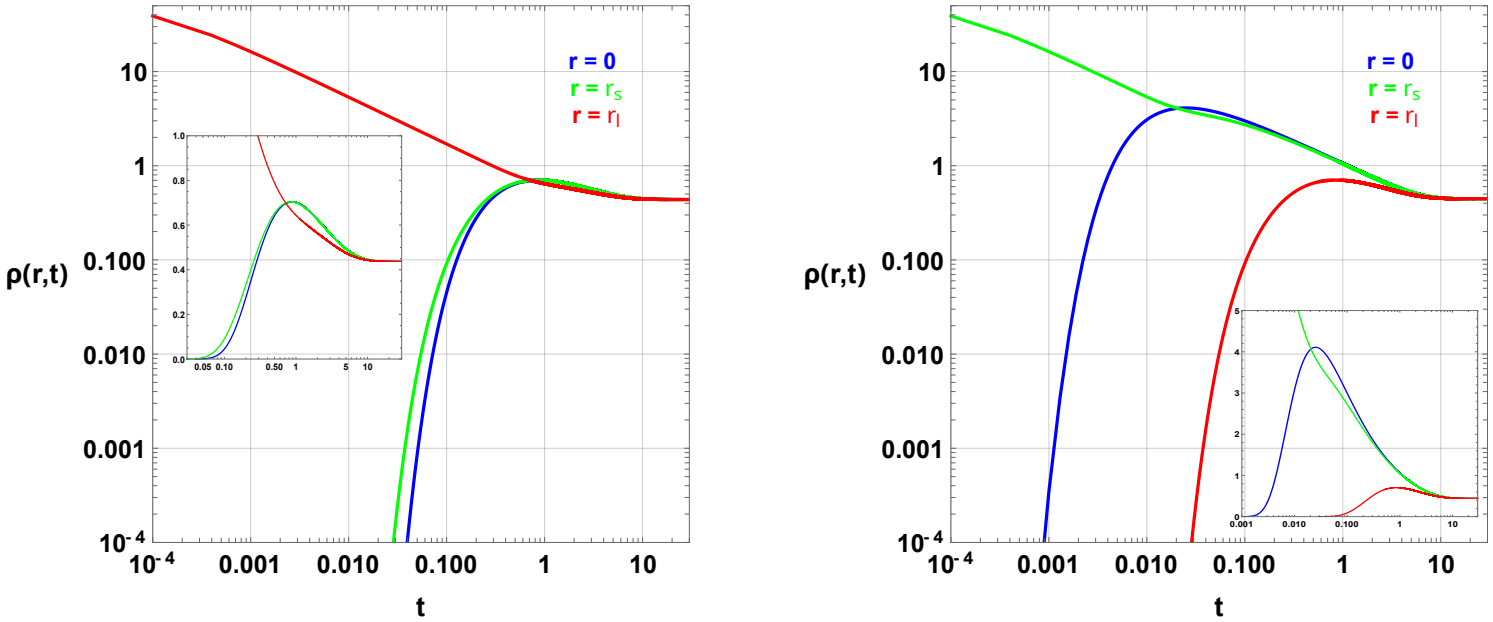
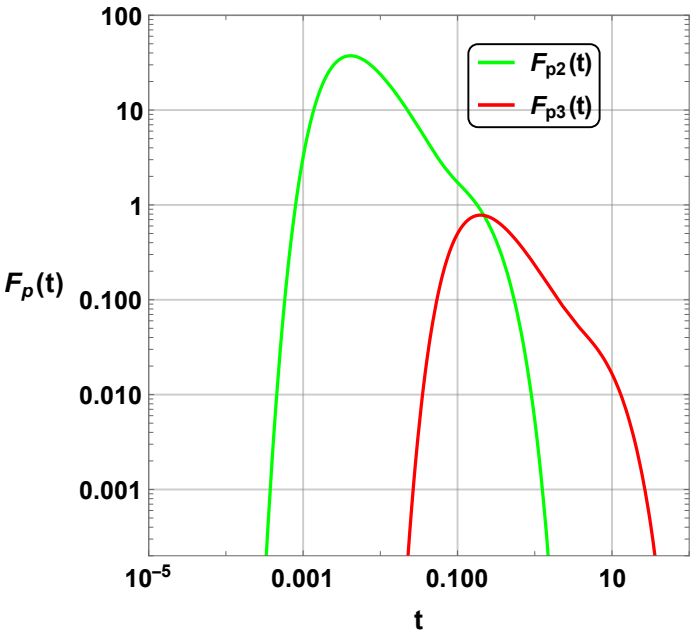


- First passage time is defined as the time required for the state of the black hole to reach an unstable black hole phase, represented by the peak of the free energy. The distribution of first passage times by $F_{p_{(ij)}}(t)$

$$F_{p_{(ij)}}(t) = -\mathcal{A}\frac{d}{dt}\int_{r_i}^{r_j}\rho(r,t)dr, \tag{10}$$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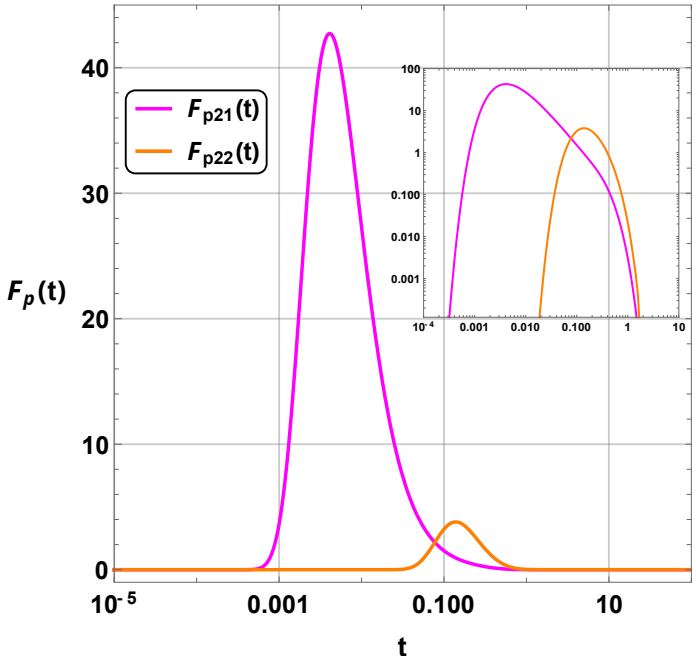
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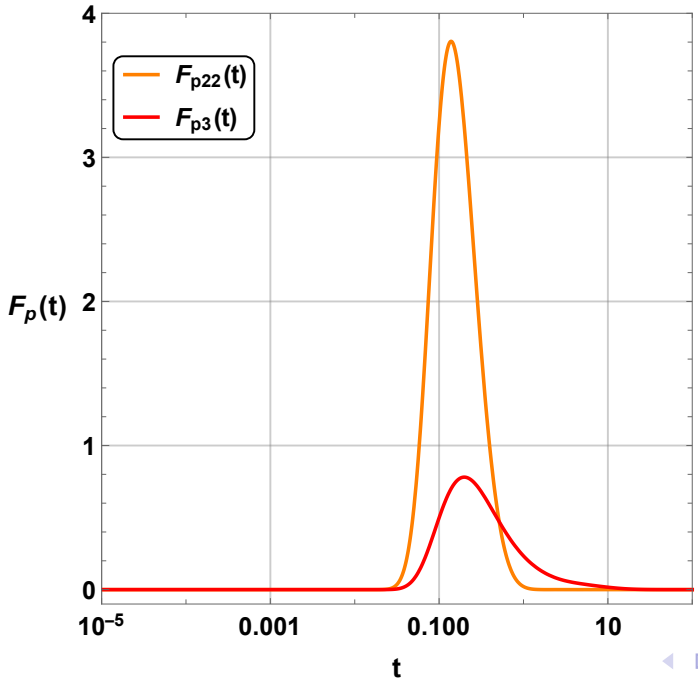
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$F_{p3}(t)$
10^{-5}
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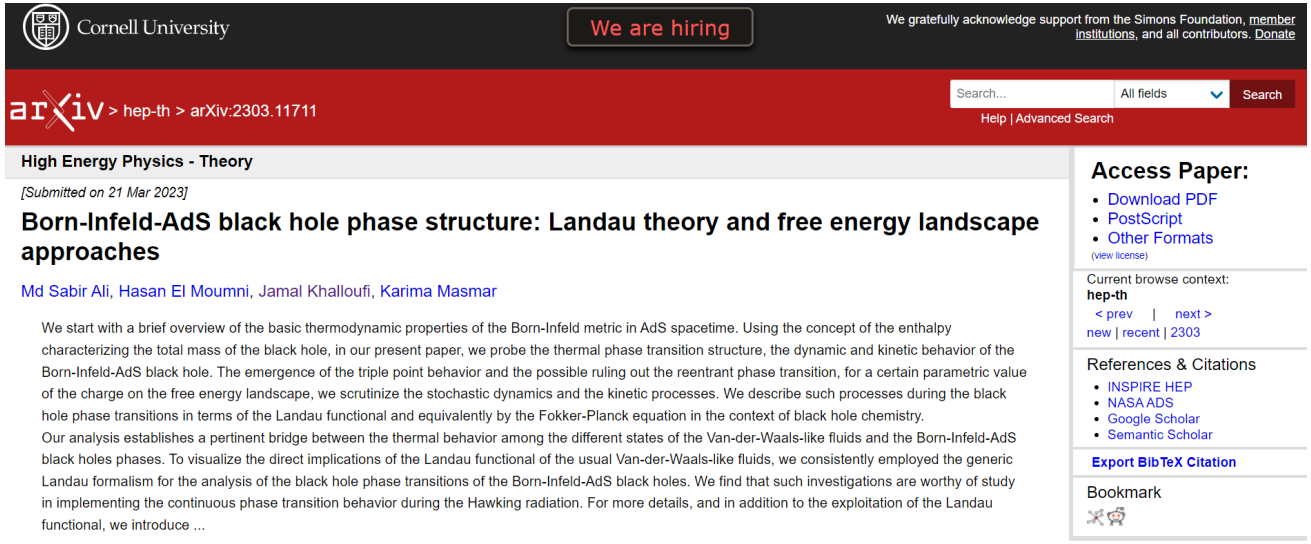
- We have determined the first passage time for the kinetic evolution of different black hole states, particularly for the triplpoint setup.

For more details :



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High Energy Physics - Theory

[Submitted on 21 Mar 2023]

Born-Infeld-AdS black hole phase structure: Landau theory and free energy landscape approaches

Md Sabir Ali, Hasan El Moumni, Jamal Khalloufi, Karima Masmar

We start with a brief overview of the basic thermodynamic properties of the Born-Infeld metric in AdS spacetime. Using the concept of the enthalpy characterizing the total mass of the black hole, in our present paper, we probe the thermal phase transition structure, the dynamic and kinetic behavior of the Born-Infeld-AdS black hole. The emergence of the triple point behavior and the possible ruling out the reentrant phase transition, for a certain parametric value of the charge on the free energy landscape, we scrutinize the stochastic dynamics and the kinetic processes. We describe such processes during the black hole phase transitions in terms of the Landau functional and equivalently by the Fokker-Planck equation in the context of black hole chemistry.
Our analysis establishes a pertinent bridge between the thermal behavior among the different states of the Van-der-Waals-like fluids and the Born-Infeld-AdS black holes phases. To visualize the direct implications of the Landau functional of the usual Van-der-Waals-like fluids, we consistently employed the generic Landau formalism for the analysis of the black hole phase transitions of the Born-Infeld-AdS black holes. We find that such investigations are worthy of study in implementing the continuous phase transition behavior during the Hawking radiation. For more details, and in addition to the exploitation of the Landau functional, we introduce ...

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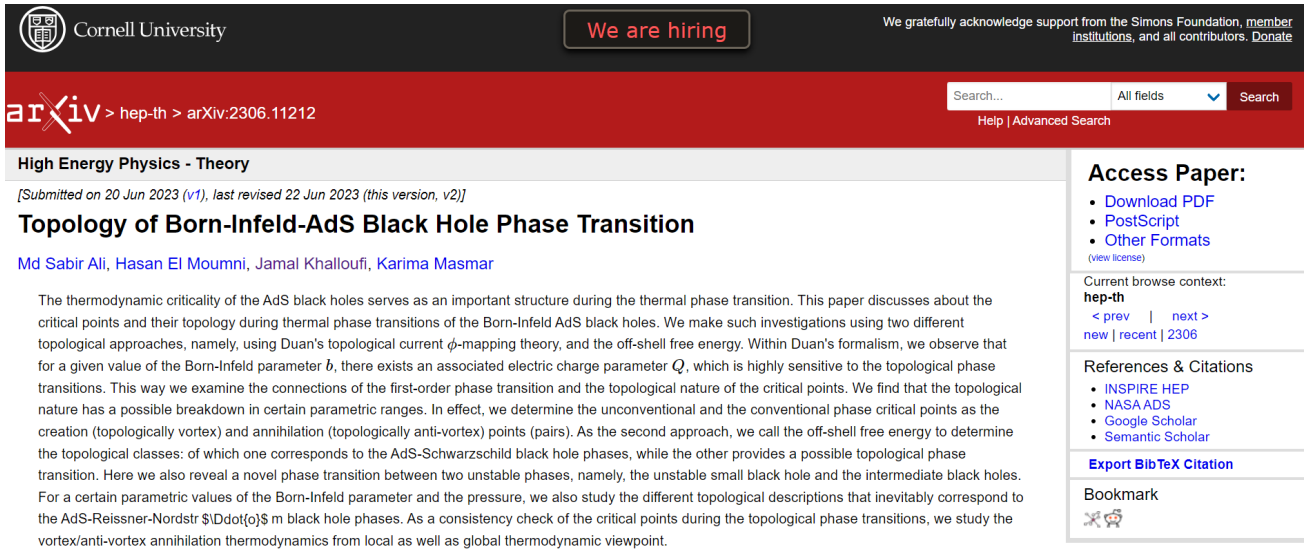

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High Energy Physics - Theory

[Submitted on 20 Jun 2023 (v1), last revised 22 Jun 2023 (this version, v2)]

Topology of Born-Infeld-AdS Black Hole Phase Transition

Md Sabir Ali, Hasan El Moumni, Jamal Khalloufi, Karima Masmar

The thermodynamic criticality of the AdS black holes serves as an important structure during the thermal phase transition. This paper discusses about the critical points and their topology during thermal phase transitions of the Born-Infeld AdS black holes. We make such investigations using two different topological approaches, namely, using Duan's topological current ϕ-mapping theory, and the off-shell free energy. Within Duan's formalism, we observe that for a given value of the Born-Infeld parameter b, there exists an associated electric charge parameter Q, which is highly sensitive to the topological phase transitions. This way we examine the connections of the first-order phase transition and the topological nature of the critical points. We find that the topological nature has a possible breakdown in certain parametric ranges. In effect, we determine the unconventional and the conventional phase critical points as the creation (topologically vortex) and annihilation (topologically anti-vortex) points (pairs). As the second approach, we call the off-shell free energy to determine the topological classes: of which one corresponds to the AdS-Schwarzschild black hole phases, while the other provides a possible topological phase transition. Here we also reveal a novel phase transition between two unstable phases, namely, the unstable small black hole and the intermediate black holes. For a certain parametric values of the Born-Infeld parameter and the pressure, we also study the different topological descriptions that inevitably correspond to the AdS-Reissner-Nordstr $\Ddot{o}$ m black hole phases. As a consistency check of the critical points during the topological phase transitions, we study the vortex/anti-vortex annihilation thermodynamics from local as well as global thermodynamic viewpoint.

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